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A subband adaptive filter architecture offers the possibility of performing the equivalent task of a fullband adaptive filter but with several key benefits. Results in the application of acoustic echo canceling demonstrate better convergence with fewer computations using a simulated acoustic echo path with white noise input. We are currently investigating performance using speech input. We are also looking at adaptive wavelet architectures for auto-regressive processes.

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Annual Technical Report II

Analysis of Multirate Adaptive Filters in Subbands

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I. Introduction

Wavelets have been the subject of a large amount of research during the past few years. As the research has developed from the initial study of the properties of wavelets into the area of their application, it is natural to start exploring the notion of "adaptive wavelets," or the possible role of wavelets in adaptive systems. This report first provides a brief overview of the basic concepts in wavelet decomposition. We then introduce the problem of finding the wavelet coefficients of the autoregressive (AR) coefficients of a time-varying signal, and propose an adaptive method.

II. Signal Decomposition via Wavelets

The Discrete Wavelet Transform (DWT), also known as Discrete Wavelet Decomposition, is a multiresolution/multirate analysis used to decompose a signal into successive layers at coarser resolutions plus detail signals at each resolution [1]. Thus a signal may be represented as a sum of two functions, one representing the coarse approximation based on a lowpass approximation of the signal, and the second representing the detail signal based on a highpass approximation. For one level of decomposition, this would be:

$$\begin{aligned} f(t) &= \Sigma\, c_{1,n}\, \Phi_{1,n}(t) + \Sigma\, d_{1,n}\, \Psi_{1,n}(t) \\ &= f_{1,v} + f_{1,w} \end{aligned}$$

where Φ represents a lowpass filter and is called the *scaling function*, and Ψ represents the highpass filter, and is known as the *wavelet*.

The outputs of a wavelet decomposition are coefficients of the coarse signal at the current level ($c_{j,n}$, where j is the level and n is the index of the coefficient) and the coefficients of the detail signal ($d_{j,n}$). These coefficients are given by:

$$c_{j,n} = \langle f_{j,v}\, ,\, \Phi_{1,n} \rangle$$

$$dj{,}n = \langle f_{j,w}\, ,\, \Psi_{1,n} \rangle$$

The filter bank formed by the highpass and lowpass filters must be a Quadrature Mirror Filter (QMF) and must form a Perfect Reconstruction Filter Bank (PRFB). Therefore, if the discrete lowpass scaling function is implemented as the FIR filter with impulse response $h_0(n)$, then the highpass wavelet filter $h_1(n)$ must satisfy [1]:

$$h_1(n) = (-1)^{n+1} h_0(N-1-n)$$

where N is the length of the filter (which must be even).

The implementation of the DWT can be performed with a fast algorithm known as *multiresolution pyramid decomposition* [2]. To compute the coefficients c and d at the first level, the signal is passed in parallel through the highpass and lowpass filters; then each output is decimated by two, since the frequency band has been cut in half for each signal. The next level of decomposition is attained by performing the same operation on the lowpass output from the previous level. Figure 1(a) diagrams three levels of this decomposition for a signal $x(n)$, and the corresponding synthesis operation for reconstructing the signal is shown in Figure 1(b). Note that the successive filtering and decimation results in an *octave band* or *constant relative bandwidth* filter bank (or subbands), as shown in Figure 2(a).

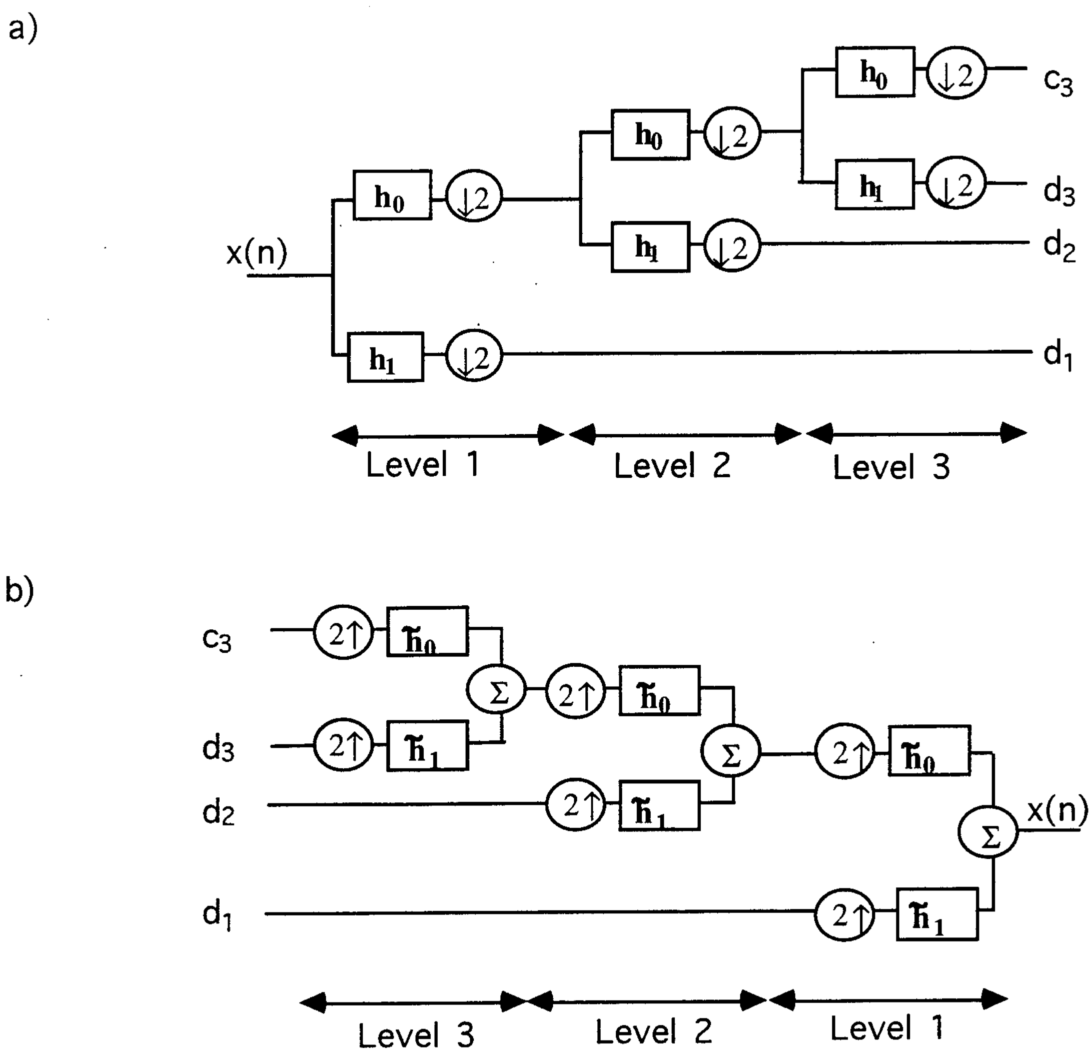

Figure 1. (a) A 3-level pyramidal decomposition filter bank and (b) the corresponding synthesis bank.

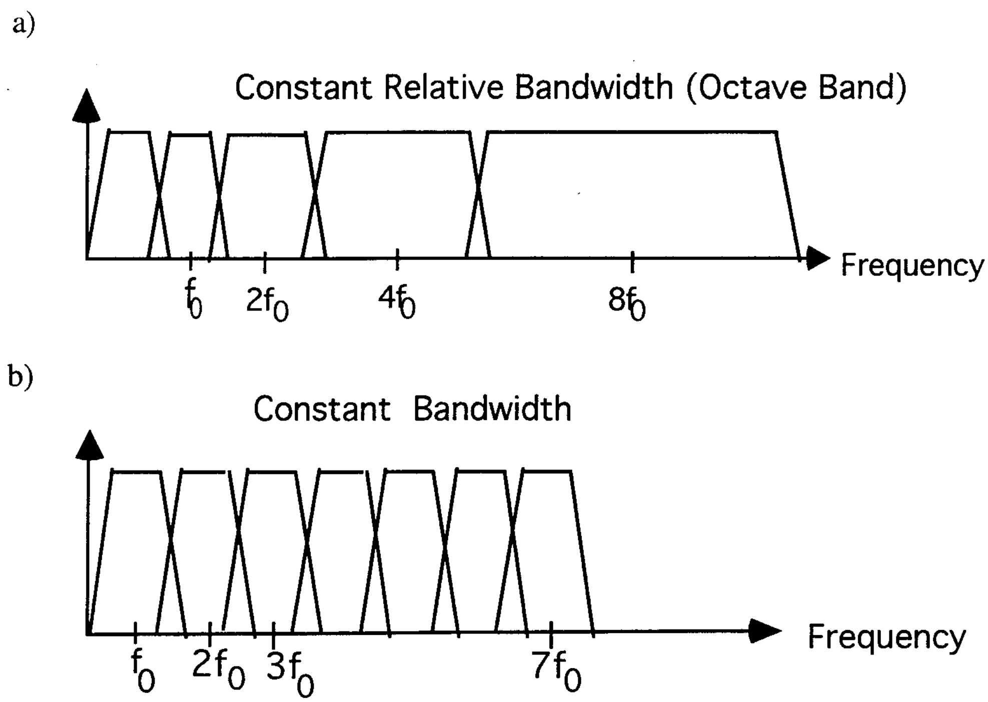

Figure 2. (a) Division of the frequency domain for the wavelet transform, to a decomposition depth of 4. (b) Division of the frequency domain for the STFT.

Wavelet decomposition has great potential in time-frequency analysis. It has been shown that the wavelet transform utilizes superior time localization at high frequencies (due to the lower frequency bands having a higher decimation factor [see Figure 1(a)]), and superior frequency localization at low frequencies (due to the narrower bandwidth for lower frequencies [see Figure 2(a)]). This is shown by the time-frequency plane tiling in Figure 3(a). The Short-Time Fourier Transform (STFT), which is a commonly used method of time-frequency analysis, uses the uniform tiling of the time-frequency plane in Figure 3(b) based on a *constant bandwidth* filter bank, shown in Figure 2(b), which does not allow for the superior localization capabilities of the wavelet decomposition. This feature of wavelets makes them particularly suited to system modeling and event detection applications.

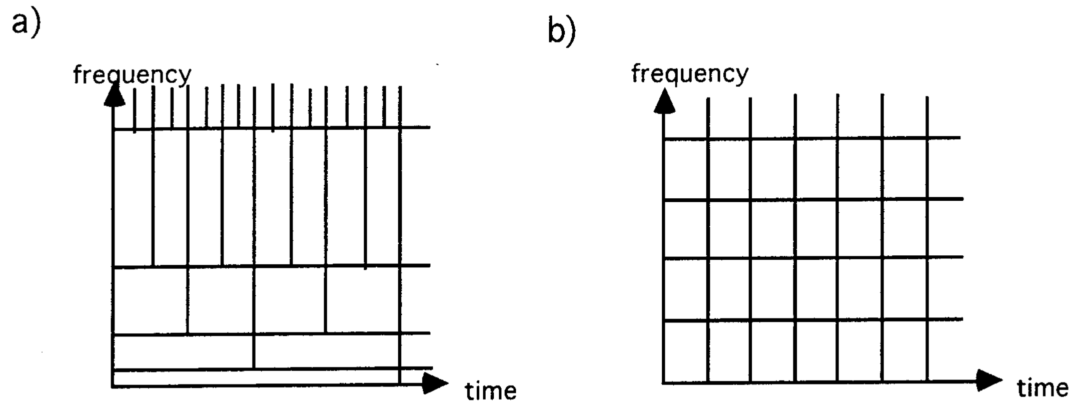

Figure 3. Time-frequency plane tiling for (a) wavelets, and (b) the STFT.

III. Modeling an AR Process Using Adaptive Wavelets

If one models a time-varying AR process by expanding the AR coefficients using a wavelet basis, the resulting system is severely underdetermined, as we will show. We propose a method to find the wavelet coefficients using an adaptive structure.

The AR/Wavelet Model

Let $x(n)$ be a discrete AR process. Then if the process is time-varying, the AR coefficients $a(n;k)$ will depend on n, and the process is modeled by:

$$x(n) = \sum_{k=1}^{p} a(n;k)x(n-k) + e(n) \tag{1}$$

where p is the AR order. The problem we wish to address is finding the optimal AR coefficients $a(n;k)$ for modeling this time-varying process.

If $x(n)$ were stationary or slowly varying, one approach would be to find the $a(n;k)$ using an adaptive method. However, the weakness with using standard adaptive techniques for tracking rapidly varying coefficients is that if the system is varying too quickly in relation to the convergence time of the adaptive algorithm, the algorithm will not converge. If, however, we view each of the AR coefficients as a signal, we can expand these signals using an orthonormal basis. The resulting coefficients of this expansion will be time-invariant, thus the convergence is not an issue.

In [5], Tsatsanis and Giannakis propose using a wavelet basis to decompose the coefficients $a(n;k)$ with respect to an orthonormal basis. They show the development for the following wavelet expansion for each $a(n;k)$:

$$a(n;k) = \sum_{m} c_{J_{max},m}^{(a_t)} \tilde{h}_0^{(J_{max})}(n - 2^{J_{max}} m) + \sum_{j=1}^{J_{max}} \sum_{m} d_{j,m}^{(a_t)} \tilde{h}_1^{(j)}(n - 2^{j} m) \tag{2}$$

where $h_0(n)$ and $h_1(n)$ are the discrete scaling function and wavelet, respectively. J_{max} is the depth of the wavelet expansion, the c's are the scaling function coefficients, and the d's are the wavelet coefficients. A two-level decomposition and synthesis is shown graphically in Figure 4.

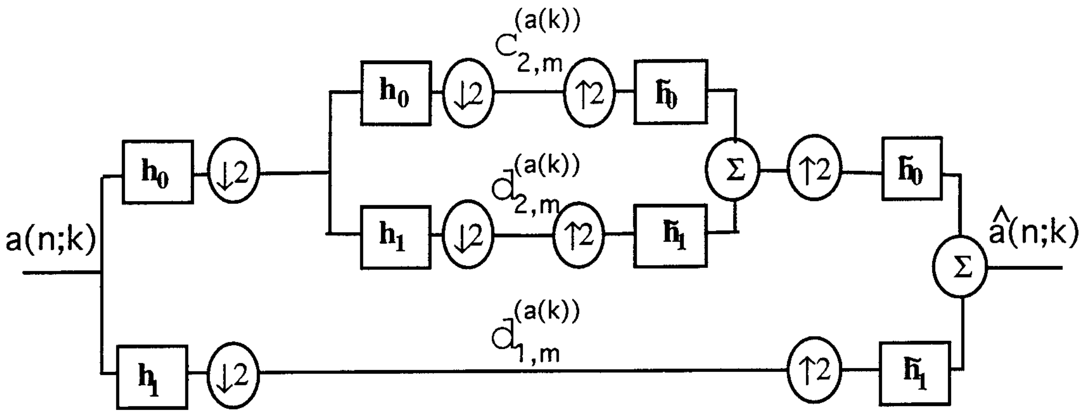

Figure 4. Decomposition and synthesis of a(n;k) to a depth of $J_{max} = 2$.

Substituting (2) into (1) results in the following expansion for *x(n)*:

$$x(n) = \sum_{k=1}^{p} \sum_{m} c_{J_{max},m}^{(a_k)} [\tilde{h}_0^{(J_{max})}(n - 2^{J_{max}} m) x(n-k)] + \sum_{k=1}^{p} \sum_{j=1}^{J_{max}} \sum_{m} d_{j,m}^{(a_k)} [\tilde{h}_1^{(j)}(n - 2^{j} m) x(n-k)] + e(n) \tag{3}$$

This is a massively underdetermined set of equations for *c* and *d* unconstrained. Tsatsanis and Giannakis propose a method for setting most of the coefficients to zero in order to obtain a system of equations which can be solved.

We propose leaving all wavelet coefficients unconstrained, and using an adaptive approach to find the optimal solution. We are currently developing adaptive techniques for this process.

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